First Order Range Calculation of ^{211}Po Produced via Multinucleon Transfer Reaction Using $^{238}\mathrm{U} + {}^{209}\mathrm{Bi}$

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Abstract

We have investigated the track of ^{211}Po within target and stopping medium produced by $^{238}\mathrm{U} + {}^{209}\mathrm{Bi}$ multinucleon transfer (MNT) reaction. Using pySRIM library, we automate the calculations given by SRIM in order to get the energy loss rate (stopping power), (-dE/dx). This function is then used to get the total energy loss of the ion (^{238}U) right before colliding with the target (^{209}Bi), in different places inside the target. Finally, we tested whether or not the product (^{211}Po) will be stopped within the Cryogenic Stopping Cell (CSC) at Fragment Separator Ion Catcher (FRS-IC) at GSI by calculating the projected range after exiting the target.

1. Introduction

MNT reactions have been used to produce neutron rich nuclei, most notably to study exotic, short-lived nuclei, and to aid astrophysical models in the explanation of rapid neutron capture (r-process) [1], which is why they are crucial to research in modern nuclear physics. We are interested in "catching" the reaction product, and for that we need to know the range and the direction of the said product.
Since precise calculations are computationally time-consuming, approach covered in this work serves as an first-order approximation in order to estimate the ranges and route said calculations into the right direction.
In this work, we studied the $(\Delta N,\ \Delta Z) = (-1,\ -1)$ MNT reaction

$$^{238}\mathrm{U} + {}^{209}\mathrm{Bi} \rightarrow {}^{211}\mathrm{Po} + {}^{236}\mathrm{Pa}$$

The product of interest is ^{211}Po, which moves through the rest of the target, where it loses energy.
After exiting the target, ^{211}Po, is supposed to be stopped within the CSC, a cyllinder ($r = 13.35,\ z = 50$) cm filled with ^{4}He of areal density of 5 mg/cm^2 which corresponds to volume density of ρ =0.1 mg/cm^3. We aim to stop ^{211}Po within the ^{4}He, i.e it needs sufficiently low energy not to hit the walls of the CSC. According to results from GRAZING calculation performed in [2], however, lower eneriges of the product correspond to larger angles, which requires caution regarding the range calculations given the small radius of the cell.

2. Calculations

The target of thickness w was discretizated into N equal strips, and inside of each we assumed constant dE/dx, i.e linear energy loss.
Total energy loss can be calculated as a sum of the contributions from every strip, i.e

$$\Delta E = \int_0^w \left(-\frac{\mathrm{d}E}{\mathrm{d}x}\right) \mathrm{d}x \approx \sum_{i=1}^{N} \left(-\frac{\mathrm{d}E}{\mathrm{d}x}\right)_i \delta x \qquad (1)$$

where (-dE/dx)$_i$ is the value of energy loss rate at the i-th strip, calculated by SRIM [3], and δx is the width of the strip, defined by

$$\delta x \equiv \frac{w}{N} \qquad (2)$$

In order to calculate the value of (-dE/dx) at a certain strip, we need the ion energy at the previous strip, which is trivial, given the linearity of the function inside every strip:

$$E_k = E_{k-1} - \left(-\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{k-1} \delta x \qquad (3)$$

where $k = 1$ corresponds to the incident beam energy (E_0). This recursion was used in automating the calculations for every strip using pySRIM library for Python. We confirmed the validity of this method by numerically integrating the function and subtracting it from the initial value, getting excellent alignment with the energy in the final strip.
Finally, given that SRIM calculates (-dE/dx) values with three significant figures, one needs to be cautious about setting the resolution too high. It is, to an extent, possible to calculate the error caused by SRIM's rounding of energies by summing the absolute difference of the energies obtained by using (3) and the values for which SRIM gave loss rates. We can hence define the error function

$$\Delta \equiv \sum_{i=1}^{N} \left| (\varepsilon_i - \varepsilon_{i-1}) + \left(-\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{i-1} \delta x \right| \qquad (4)$$

where ε_i corresponds to energy for which SRIM calculated (-d$E/$d$x)_i$.

3. Results

We considered the incident ^{238}U beam with the energy of $E_0 = 2.8$ GeV entering the 30 μm thick ^{209}Bi target. Assuming the reaction occured at a distance x_0 within the target, we considered two cases: close to the beginning of the target ($x_0 = 5$ μm), and close to the middle ($x_0 = 10$ μm). We found the Bragg's curves for both cases, expressed in MeV$/\mu$m^2. Using data from the plots, we calculated the energy of ^{238}U right before the reaction happened (i.e the loss caused by travelling through ^{209}Bi, as well as the exit energy of ^{211}Po. We also make use of the results provided in [2] regarding the energy of ^{211}Po right after the reaction, as well as the angle relative to the incident beam. Results for the energy loss rate for the two aforementioned cases are presented in Figure 1. Figure 2 shows how energy and direction of ^{211}Po change depending on where the reaction occured. We observe almost linear fall of the product energy with distance.

Using the method defined in (4), we obtain the following errors for the two cases we considered:

$$\begin{aligned} \Delta_5 &= 39.5 \text{ MeV} \\ \Delta_{10} &= 56.4 \text{ MeV} \end{aligned} \tag{5}$$

Where Δ_5 and Δ_{10} correspond to the errors for reaction which occured at $x_0 = 5$ μm and $x_0 = 10$ μm, respectively.

Close to the Coloumb barrier cutoff ($x_0 = 15$ μm), we found that ^{211}Po will hit the CSC, considering its creation at lower energies corresponds to larger angles (specifically, the range is 24.6 cm, at the angle of 40°). If the the target thickness is set to 50 μm, ^{238}U beams with energies as high as 3 GeV will not be able to create sufficiently fast ^{211}Po to enter the CSC. If the target was, however, thinner (20 μm), ^{211}Po will be faster than necessary, causing it to collide with the walls of the CSC (for 3 GeV beam, ^{211}Po is projected to go as far as 70 cm at 28° angle, which clearly exceeds the dimensions of the CSC).

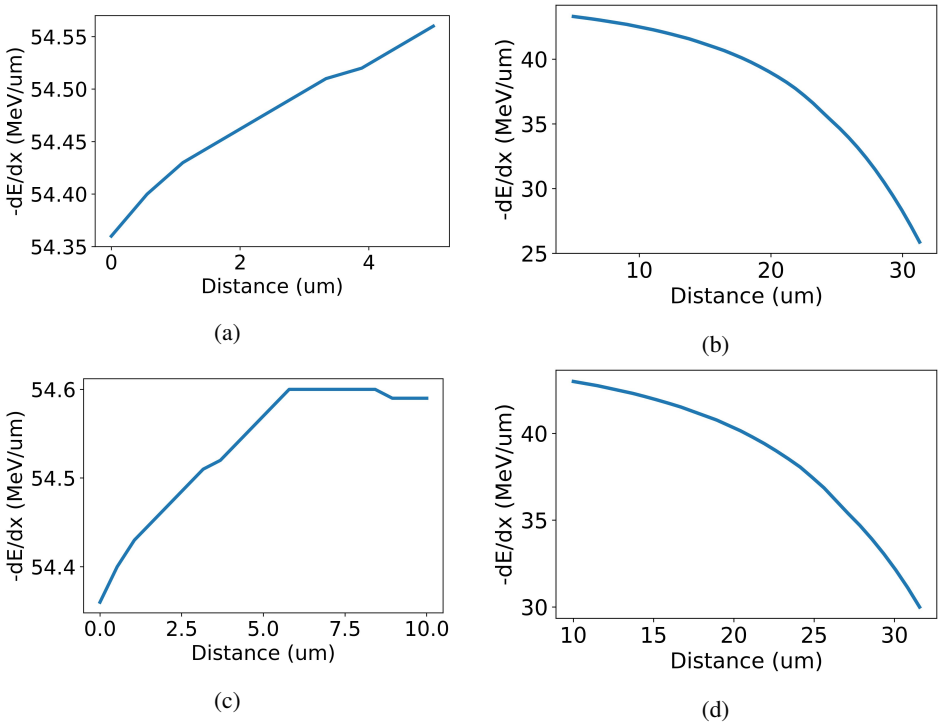
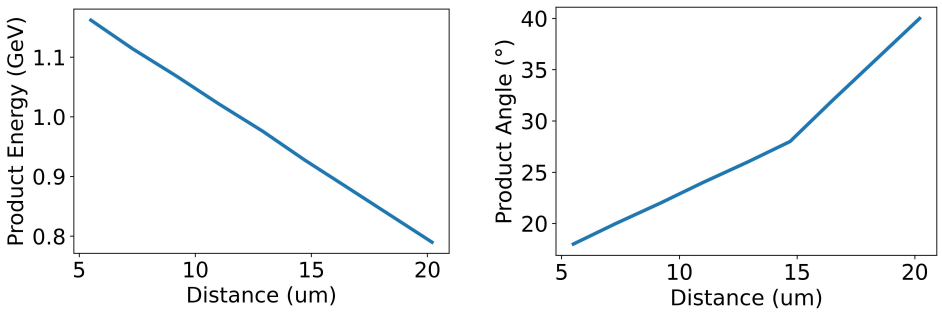


Figure 1: (-d$E/$dx) as a function of distance travelled within the target (also known as Bragg curve) for $x_0 = 5$ μm for ^{238}U (before the reaction) (a), and ^{211}Po (after the reaction) (b). Energy of ^{238}U drops to 2528 MeV, and upon reaction ^{211}Po is created with the energy of 1162 MeV, and it travels with the angle of 18° relative to the incident beam. It exits the target with 168 MeV, going 25.1 cm into the CSC and stopping before hitting the walls. Calculations are replicated for $x_0 = 10$ μm, and energy loss rate is shown again for ^{238}U (c), where we observe Bragg peak at 5.3 μm due to the larger distance travelled and ^{211}Po (d). Assuming reaction happened here, ^{211}Po will have 229 MeV at the end of the target, capable of going 29.6 cm inside ^{4}He. Again, it is expected to be stopped within the CSC, since it will travel with 22° angle.



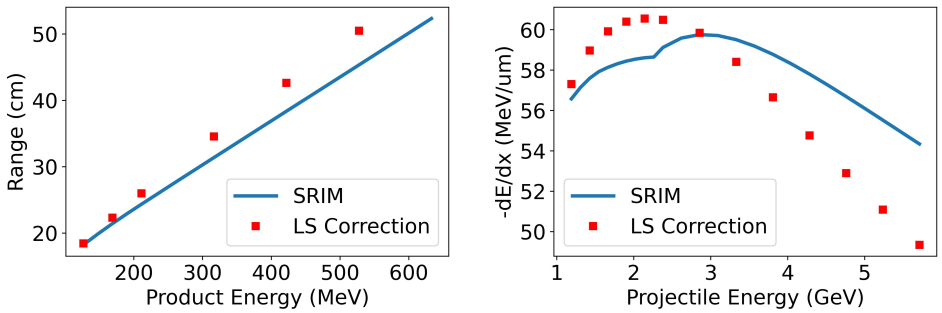
(a) (b)

Figure 2: Dependance of energy (a) and angle (b) of ^{211}Po created. We used the results from GRAZING code, as presented in [2]. Cutoff is created at 20.2 μm, since energy of ^{238}U will drop below the Coloumb barrier further into the target, and no reaction will happen.

4. Conclusion

Evidently, optimal target thickness for stopping ^{211}Po inside the CSC is 30 μm. There is a solid window of 5 μm inside the target where reactions could occur with corresponding ions being stopped inside the CSC. Using thicker or thinner target results in ^{211}Po being stopped inside the target and hitting the walls of the CSC, respectively. More freedom in system configuration (projectile energy and target thickness) would be granted by increasing the density of ^{4}He, as well as making the diameter of the stopping cell larger.

Finally, thanks to the large energies of the nuclei, there's no need to introduce corrections such as those proposed in [4]. At GeV scales, LS theory and results given by SRIM show little disrepancy, as shown in Figure 3.



(a) (b)

Figure 3: Comparison of SRIM and LS theory in calculating the range of ^{211}Po in ^{4}He (a) and in the (-dE/dx) (b). We observe good alignment of the models in the window of energies we worked with.

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